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Mutual influence of the faces contact area and the pre-fracture zone near the tip of the interfacial crack

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Abstract.
BACKGROUND: Under plane strain conditions, a crack model was developed on a plane interface between two different materials, which assumes the existence near its tip of the faces contact area and a narrow lateral pre-fracture zone in a less crack-resistant material of the composite compound. The pre-fracture zone is modeled by the line of normal displacement rupture, on which the normal stress is equal to the tensile strength of the material. Assuming that the dimensions of the pre-fracture zone and the contact zone have the same order of magnitude and are significantly smaller than the crack length, the problem is reduced to the vector Wiener–Hopf equation.
METHODS: An approximate method for solving the vector Wiener–Hopf equation was developed, which was used to obtain the equations for determining the sizes of the pre-fracture zone and the contact faces area. The pre-fracture zone orientation was determined from the condition of the potential energy maximum accumulated in the zone. Numerical calculations of the indicated parameters and analysis of their dependences on the configuration and module of external load are executed.
RESULTS: A significant mutual influence of the pre-fracture zone and crack faces contact on their sizes and orientation of the zone was revealed.

Keywords: Interfacial crack, contact zone, pre-fracture zone, vector Wiener–Hopf equation, approximate method

1. Introduction

The investigations of cracks located on the interface of different materials (interfacial cracks) discovered many factors, which essentially influence a stress-strain state in the vicinity of their tips. In particular, since the classic model of interfacial crack-cut led to physically incorrect spatial oscillations of the displacement of the crack faces with a mutual intersection at the approach to its tip (e.g. [7,22,23,41,46,49]), an alternative model of interfacial crack was proposed to eliminate them, which assumes the contact of the crack faces at the part adjacent to the tip (e.g. [2,10–13,25,40,43]). At the same time, the concentration of stresses at the tips causes the formation of pre-fracture zones both in the joined materials and in the interfacial layer. In accordance with the conclusions of experimental investigations [31,32,38] in each of the pre-fracture zones in a small part adjacent to the tip, the destruction of material occurs, which is accompanied by the breaking of interparticle bonds and a significant increase of relative deformations. Besides due to the concentration of stresses at approaching the tip, there is a significant increase in the compressive stress in the contact faces area, which makes the faces relative shear impossible and leads to the formation of the stick area in the contact zone part adjacent to the tip of the crack. When modeling an interfacial crack, it is also advisable to take into account the interface non-ideality, which consists of the presence of thickness and elastic properties of the connecting layer [26–29]. We also note the absence of symmetry in a piece-wise homogeneous body, which does not permit to separate the symmetric and skew-symmetric parts with the corresponding stress intensity factors of I and II modes in the solution of an elastic problem about an interfacial crack.

The multifactorial nature of the problem of interfacial crack prevents the obtaining of a complete and rigorous description of the stress-strain state near its tips. To date, only a few attempts have been made that take into account several of the above-mentioned factors simultaneously. Dorogoy and Bank-Sills [15,16] performed a numerical calculation of the sizes of the contact and stick areas of the crack faces. Shih and Asaro [47,48] used numerical methods to analyze the plastic zones near the tip of the interfacial crack in both joined materials. Antipov et al. [3] considered an interfacial crack under the conditions of an anti-plane shear for the case of a non-ideal interfacial joint, simulating it by effective contact conditions involving a jump in displacements at the interface. Kim et al. [37] investigated the effect of the connecting layer elasticity on the interfacial crack and revealed the disappearance of the oscillatory singularities at the crack tips.

In [17–19,30] a number of analytical models of the interfacial crack with partial contact of the faces were investigated in the presence in one of the materials a small-scale pre-fracture zone containing a small zone of destruction. Since modern mathematical methods do not allow to obtain analytical solutions of the problems of nonlinear fracture mechanics about calculating the shape and size of a pre-fracture zone, the authors used an approximate model of the zone based on the Dugdale–Leonov–Panasyuk model [20,39], representing the zone as the line of discontinuity of displacements, on which the stresses satisfy one or another condition for the transition to the pre-fracture state depending on the properties of the material (for example, brittle or ductile). These models were implemented by the authors for the case of prevailing shear loading, when the length of the contact zone is significantly larger than the length of the small-scale pre-fracture zone, and in the case of prevailing tensile loading, when the length of the small-scale pre-fracture zone is significantly greater than the length of the contact zone. These simplifications made it possible to reduce the corresponding problems to the scalar Wiener–Hopf equations, which are solved analytically exactly. However, the assumptions adopted in these works make it impossible to investigate the mutual influence of the faces contact area and of the pre-fracture zone on their sizes.

In this article we consider the problem of calculating the parameters of the commensurate pre-fracture and contact zones near the tip of the crack located on a plane interface between the two parts of a piecewise-homogeneous brittle body with different mechanical properties. Using the Mellin integral transform the problem is reduced to the vector Wiener–Hopf equation, which is solved approximately using a specially developed iterative method. Despite the schematic nature of the used model of the pre-fracture zone, the study allows one to make quantitative estimates and qualitative conclusions about the mutual influence of the pre-fracture zone and the faces contact zone on their parameters.

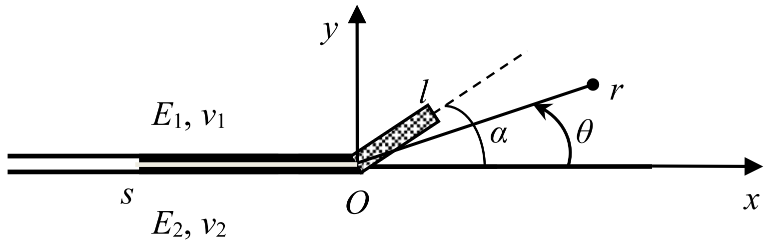

Fig. 1. Computational model of the structure of the near-tip area of an interfacial crack with the faces contact and the lateral pre-fracture zone.

2. Statement of the problem

Under plane strain conditions, we consider the problem about the structure of an area near one of the tips of a crack located on a plane interface between two different homogeneous isotropic materials with Young's modulus E_1 and E_2 and Poisson's ratios ν_1 and ν_2. We investigate the case when the configuration and magnitude of the external load and the parameters of the joined materials are such that they ensure the presence at the tip of the crack of a contact faces area and a lateral pre-fracture zone in a less crack-resistant material (for definiteness in a material with elastic constants E_1, ν_1) with linear sizes of the same order of magnitude much less than the length of the crack and the characteristic sizes of the investigated body. According to the localization hypothesis [44], the pre-fracture zone in a brittle material is modelled by a straight line of the normal displacements discontinuity propagating from the crack tip at an angle α to the interface of the materials. On the line of discontinuity, the normal stress is equal to the tensile strength of the first material σ_1. The contact zone is simulated by a notch whose faces interact according to the Coulomb law of dry friction with a friction coefficient μ. We also assume that compressive normal stress acts on the contact zone faces and that solely the tangential component of displacements may have jumps here.

Since the length s of the contact zone and the size of the pre-fracture zone l are much smaller than the crack length L, and the stress-strain state is determined only in the vicinity of the tip, this enables us to consider the body as a piecewise-homogeneous plane containing a semi-infinite notch on the interface and a line of discontinuity of finite length originating at the tip and propagating into the first material at an angle of α to the interface (Fig. 1). A part of the notch adjacent to the tip is in contact with friction.

The purpose of the problem is to determine the sizes of both zones and the orientation of the pre-fracture zone. Assuming that outside the contact zone the crack faces are free of loads and taking into account the modeling of the pre-fracture zone by a segment of line of the discontinuity of normal displacements, we arrive at the static boundary value problem of the theory of elasticity with the following boundary conditions:

$$\theta = 0, \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_\theta \rangle = \langle u_r \rangle = 0;$$

$$\theta = \alpha, \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_r \rangle = 0;$$

$$\theta = \pm\pi, \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_\theta \rangle = 0, \quad \tau_{r\theta} = -\mu\sigma_\theta; \tag{1}$$

$$\theta = \alpha, \quad r < l, \quad \sigma_\theta = \sigma_1; \quad \theta = \alpha, \quad r > l, \quad \langle u_\theta \rangle = 0; \tag{2}$$

$$\theta = \pm\pi, \quad r < s, \quad \langle u_\theta \rangle = 0, \quad \tau_{r\theta} = -\mu\sigma_\theta; \quad \theta = \pm\pi, \quad r > s, \quad \sigma_\theta = \tau_{r\theta} = 0, \tag{3}$$

where $\langle f \rangle$ is a jump of f.