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INVESTIGATION OF THE PROCESS ZONE NEAR THE TIP OF AN INTERFACE CRACK IN THE ELASTIC BODY IN SHEAR WITHIN THE FRAMEWORK OF THE COMPLEX MODEL

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Under the conditions of plane deformation, by using the Wiener–Hopf method, we perform the numerical analysis of a narrow small-scale process zone in the elastic body subjected to shear near the tip of an interface crack whose faces are in contact with friction. The process zone originates at the crack tip and makes a certain angle with the interface of the media. It is modeled by the straight line of discontinuities of displacements and consists of two segments. In the segment adjacent to the crack tip, which models the region of destruction of the material, both normal and tangential displacements have discontinuities. At the same time, in the second segment, only the normal component of displacements suffers discontinuities. The angle between the process zone and the interface of the media is determined from the condition of maximum of circumferential tensile stresses. We determine the sizes of the entire process zone and the region of destruction, the crack-tip opening displacements, and maximum opening displacement of the initial process zone. We also investigate the influence of the friction coefficient on the parameters of the process zone. By using the deformation criterion of fracture, we analyze the role played by the zone of destruction in the onset of crack propagation. It is shown that the crack starts as a result of the relative shear of its faces near the tip. The comparison of the obtained results with the data of other researchers is presented.

Cracks located on the interface of two different media attract attention of the researchers due to the great importance of their investigation for the development of the fracture mechanics of composites, glued and welded joints, which are now extensively used in contemporary technological materials. At present, a completely satisfactory theory of interface cracks does not exist [2] because the classical model of interface cracks proposed in the works by Willams, England, Erdogan, Rice, and Sih contains a physical incorrectness in the form of spatial oscillations of displacements observed on approaching the crack tip, which leads to the intersection of its faces. However, an alternative model developed by Comninou, Dundurs, and other authors in which it is assumed that the crack faces are in contact near the tip removes the indicated oscillations but appears to be incorrect for certain configurations of loads because the length of the corresponding contact zone may be of the same order as the sizes of atoms or even smaller but, in this case, the continuum theory of elasticity becomes inapplicable. However, both these models are used in the theory of interface cracks because they lead to meaningful physical conclusions and acceptable practical results.

The application of the classical model of (open) cracks is, in our opinion, admissible in the case where tensile loads are predominant in the vicinity of the crack tip in the direction perpendicular to the crack plane, i.e. in the case where the contact model gives extremely small sizes of the region of contact of the faces [15, 21, 26]. To remove the oscillating singularity, it is proposed to take into account the nonlinear behavior of the material

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near the tip and, in particular, the formation of the so-called process zone in its vicinity [29]. Examples of determination of the process zones within the framework of some models [5, 7, 8] confirm the efficiency of this approach. However, under the conditions of shear, the sizes of the contact zone become comparable (in the order of magnitude) with the crack length [4, 15, 22–26]. Therefore, the application of the contact model becomes unavoidable. Since, in this case, the concentration of stress is preserved at the crack tip adjacent to the contact zone, the formation of the process zone should be also expected in the vicinity of the crack tip.

The determination of the parameters of the process zone is an urgent problem of fracture mechanics because, as a result, it becomes possible to describe the stress-strain state near the tip more completely and, hence, to specify the conditions of limit equilibrium of the crack more exactly. The problem of rigorous determination of the process zone as a three-dimensional region of the material with nonlinear determining equations is reduced to a complicated boundary-value problem of the theory of partial differential equations with an *a priori* unknown boundary. The solution of this problem is possible only by using numerical methods. At present, the only model of the process zone which, in many cases, admits analytic investigations is the Leonov–Panasyuk–Dugdale model and its various modifications and analogs in which the process zone is replaced by the surface of discontinuity of displacements. On this surface, one or another condition of transition into the prefracture state (depending on the properties of the material) is specified, which makes it possible to reduce the original problem to the problem of the theory of elasticity with special boundary conditions.

The results of experimental investigations demonstrate that the process zones near pointed stress concentrators in elastoplastic bodies have a complex structure and consist of a fairly developed plastic zone and a quite small region of destruction of the material adjacent to the tip of the concentrator with very high levels of strains caused by the formation of a large number of microdefects of different types [11–13]. Numerous theoretical models were proposed with an aim to take into account this fact. These are the *model of cohesive zone* that contains the zone of fracture processes [30, 31]; the *model of the process zone* in a linearly hardening material *with a zone of loosening* [17, 18]; the *complex model of the process zone with a region of destruction* [9, 10]. Despite the differences in terminology, the basic concepts of the indicated models are close and based on the development of the Leonov–Panasyuk–Dugdale model.

According to the *complex model of the process zone* formed in a bimaterial body at the end of a crack with contacting faces located on the interface of elastic media, the development of this zone occurs in two stages [9, 10]. In the first stage, a narrow initial lateral process zone forms as a result of loading of the body accompanied by the formation of the contact zone. Since its formation does not remove stress concentrations near the crack tip, in the second stage, this leads to the formation of a region of elevated strains of the material, i.e., the region of destruction, in a part of the process zone adjacent to the crack tip. The subsequent increase in the external load is accompanied by the simultaneous increase in the sizes of both zones. In the present work, their determination under the conditions of plane deformation is reduced to the solution of the following two problems:

In the first problem, by the Wiener–Hopf method, we perform the numerical analysis of the initial process zone modeled by a lateral line of discontinuity of normal displacements originating from the tip of an interface crack whose faces interact according to the law of dry friction. The conditions of loading and the configuration of the body are taken into account within the framework of linear fracture mechanics via the stress intensity factor, which is assumed to be known from the solution of one or another additional problem. Therefore, the obtained results are fairly general and applicable to a broad class of external conditions satisfying the initial assumptions. The orientation of the process zones is determined from the condition of maximum of tensile circumferential stresses σ_θ [19]. The sizes of the zone and the character of changes in the stress-strain state near the crack tip are found in the process of solution of the corresponding boundary-value problem. Similar calculations were previously performed separately within the framework of the classical model of open interface cracks [5, 8] and within the framework of the Comninou model without taking into account the possibility of friction between the contacting faces [6].