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"TRIDENT" MODEL OF PLASTIC ZONE AT THE END OF A MODE I CRACK APPEARING ON THE NONSMOOTH INTERFACE OF MATERIALS

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By using the Wiener–Hopf method, within the framework of the "trident" model, we compute a small-scale plastic zone formed under conditions of plane deformation at the corner point of the interface of two different elastoplastic materials in the presence of a mode I crack originating from this point. The indicated zone is modeled by two symmetric lateral lines of discontinuities of tangential displacements and the line of discontinuity of normal-displacement on the continuation of the crack. We deduce analytic expressions for the evaluation of the sizes of the plastic zone and crack opening displacements. On the basis of numerical investigations, we analyze the dependences of parameters of this zone on the opening angle of the interface and the elastic characteristics of the materials.

Keywords: corner point of the interface of different media, mode I crack, "trident" model of plastic zone, crack opening displacement.

The theoretical and experimental investigations demonstrate that the plastic zone formed near pointed stress concentrators in elastoplastic materials has a complex structure and contains a fairly developed plastic domain in which shear strains are predominant and a much smaller plastic process zone adjacent to the tip of the concentrator with high levels of both shear and tensile strains. With regard for these features and symmetry conditions, the "trident" model was proposed in [1] for the description of the plastic zone formed at the end of a mode I crack originating from the corner point of the interface of two media. According to this model, the zone develops in two stages. In the first stage, two narrow lateral plastic strips (modeled by segments of discontinuities of tangential displacements) symmetrically propagate from the crack tip at a certain angle to its initial direction in one of the materials of the joint. If the appearance of these strips does not remove the stress concentration near the crack tip, then, in the next stage, the plastic process zone modeled by a segment of discontinuities of normal displacements can be formed on the continuation of the crack (Fig. 1).

The numerical analysis of the plastic zone performed within the framework of the "trident" model with regard for the two stages of its development is reduced to the solution of four problems (for the slopes of plastic strips β greater and smaller than a half of the opening angle of the interface α) by the Wiener–Hopf method. A similar problem for a plastic strip on the interface of two materials was solved in [2].

Parameters of the Initial Lateral Plastic Zone

Assume that, under the conditions of plane deformation in a piecewise homogeneous isotropic elastoplastic body, a mode I crack originates from a corner point of the interface of materials into the material with Young's modulus E_1 and Poisson's ratio ν_1.

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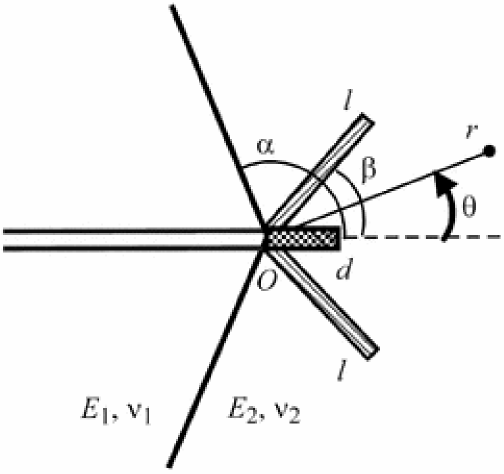

Fig. 1. “Trident” model of plastic zone.

Since the corner point of the interface of two materials playing the role of the origin of mode I crack is a stress concentrator with power singularity [3], this leads to the formation of a plastic zone in its neighborhood. We assume that, in the initial stage of development of the plastic zone, two narrow lateral plastic strips of the same length are formed in the material with elastic constants E_i and ν_i $(i = 1, 2)$. Their length is much smaller than the crack length (here and in what follows, subscript i marks the quantities corresponding to the plastic strips in the ith material). According to the hypothesis of localization [4], we model these strips by two rectilinear segments of discontinuities of tangential displacements inclined at an angle β_i to the direction of crack continuation. The tangential stresses on these segments are equal to the yield limit of the i th material τ_{is}.

We compute the lengths of the lateral plastic strips according to the Wiener–Hopf method by analogy with the problem of plastic strips in the connecting material [5, 6] and arrive at the following expression:

$$l_i = \left(\frac{|C|}{\tau_{is}}\right)^{-1/\lambda} R_i(\beta_i), \tag{1}$$

where C is a constant characterizing the intensity of external force field (it is specified by the conditions of the problem), λ in the least root, in the interval $(-1, 0)$, of the equation [3]:

$$D_0(-1-x) = 0, \tag{2}$$

$$\begin{aligned} D_0(p) = {} & 4e^2[\kappa_2 \sin 2p\alpha - p\sin 2\alpha][p^2 \sin^2 \alpha - \sin^2 p(\pi - \alpha)] \\ & + e\Big\{(1+\kappa_1)(1+\kappa_2)\sin 2p\pi - 4[\kappa_2 \sin 2p\alpha - p\sin 2\alpha][p^2 \sin^2 \alpha - \sin^2 p(\pi-\alpha)] \\ & - (p\sin 2\alpha + \sin 2p\alpha)[(1+\kappa_1)(1+\kappa_2) - 4(p^2 \sin^2 \alpha + \kappa_1 \sin^2 p(\pi - \alpha))]\Big\} \end{aligned}$$