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INVESTIGATION OF THE INITIAL STAGE OF KINKING OF AN INTERFACE CRACK AT AN ANGULAR POINT OF THE INTERFACE OF TWO MEDIA

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We present the solution of the problem of initial process zone under the conditions of plane deformation in the vicinity of an angular point on the interface of two dissimilar elastic materials playing the role of the origin of an interface crack. We study the dependences of orientation, length, and crack opening displacement on the external load, opening angle of the interface, and the elastic parameters of the joined materials. The conditions of crack initiation are analyzed by using the deformation criterion.

Keywords: angular point on the interface of two media, interface crack, process zone, crack kinking.

The last decade is marked by the appearance of numerous works devoted to the phenomenon of kinking of a crack located in the plane interface of two dissimilar media [1–5]. In these works, the main method used for the determination of the angle of kinking is based on the introduction of a short lateral cracklike branch on the continuation of the crack with the use of one of the existing criteria for the choice of the direction of propagation of this branch. However, the formation of a process zone playing the role of a stress concentrator at the end of the interface crack was not taken into account in these works. Note that the presence of this zone strongly affects the stress-strain state near the crack tip. This fact was taken into account in [6, 7], where an efficient method was proposed for the determination of the initial lateral process zone whose orientation specifies the direction of crack growth caused by the increase in the applied load.

At the same time, there are no available investigations of similar problems for interface cracks originating from angular points on the interface of two media. Their solution is of high interest for the fracture mechanics of composites with granular fillers, welded or glued joints of piecewise homogeneous wedgelike bodies, etc. The aim of the present work is to compute the initial lateral process zone at the vertex of an angle of the broken interface of two elastic media containing an interface crack, determine its orientation, length, and opening displacement, and study the conditions and direction of crack initiation.

Formulation of the Problem

Under the conditions of plane deformation, we consider the problem of the initial stage of kinking of a rectilinear interface crack of length L originating from an angular point of the broken interface of two dissimilar elastic homogeneous isotropic materials with Young's moduli E_1, E_2 and Poisson's ratios ν_1, ν_2. The opening angle of the interface is α. In this stage, according to the hypothesis of localization [8, 9], we assume that a lateral process zone is formed and propagates from an angular point in a thin layer of the less crack-resistant material of the composite (assume that this is the first material). In this connection, in view of the tensile character

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of propagation of this zone, it is simulated by the line of discontinuity of normal displacements inclined to the interface of the media at an angle β. The normal stresses in this line are equal to the resistance of the first material to separation σ_1. The length of the line of discontinuity l and its slope angle β are determined in the course of the solution of the posed problem.

At the beginning of the process of crack propagation, the length of the process zone is much smaller than the crack length and all other sizes of the body. Hence, since the stress-strain state is studied only in a certain vicinity of the process zone, the original problem is reduced to the problem of a line of discontinuity of finite length in a piecewise homogeneous plane with interface of two media in the form of the sides of an angle whose vertex plays the role of the origin of a semiinfinite crack propagating along one side of the angle.

The boundary conditions at infinity are determined from the asymptotic solution of a similar problem (but without discontinuity lines) in the vicinity of the angular point corresponding to the roots of the characteristic equation in the strip $-1 < \operatorname{Re}\lambda < 0$ [10]:

$$D(\lambda) = 0, \tag{1}$$

$$D(\lambda) = -(1+\kappa_1)^2 d_1 - 4(1+\kappa_1)(e-1) d_1 d_2 - e^2(1+\kappa_2)^2 d_3$$

$$+ 4(e-1)^2 d_1 d_3 + 4e(1+\kappa_2)(e-1) d_3 d_4 + 2e(1+\kappa_2)(1+\kappa_1) d_5,$$

$$d_1 = (\lambda+1)^2 \sin^2\alpha - \sin^2(\lambda+1)\alpha, \qquad d_2 = \sin^2(\lambda+1)(2\pi-\alpha),$$

$$d_3 = (\lambda+1)^2 \sin^2\alpha - \sin^2(\lambda+1)(2\pi-\alpha), \qquad d_4 = \sin^2(\lambda+1)\alpha,$$

$$d_5 = d_1 + \sin(\lambda+1)\alpha \sin 2\lambda\pi \cos(\lambda+1)(2\pi-\alpha), \qquad e = \frac{1+\nu_2}{1+\nu_1}\frac{E_1}{E_2}, \qquad \kappa_{1(2)} = 3 - 4\nu_{1(2)}.$$

The numerical analysis of Eq. (1) shows that the number of these roots can be equal either to two or to three. Moreover, the orders of the smallest two roots λ_1 and λ_2 are close to -0.5 and the order of λ_3 is close to 0 [11]. In addition, there exists a range of opening angles $(\alpha_{\min}, \alpha_{\max})$ in which the roots λ_1 and λ_2 are complex conjugate $(\lambda_1 = \overline{\lambda_2})$. In what follows, in finding the parameters of the process zone, we assume that the strip $-1 < \operatorname{Re}\lambda < 0$ contains only real roots or complex roots and a single real root.

Assume that the crack faces are load-free and that the conditions of perfect adhesion are realized on the interface, which guarantees that the stresses and displacements are continuous. The asymptotics corresponding to the principal terms of expansions of the solution of the problem (in the absence of the process zone) is specified at infinity. Since the process zone is modeled by a segment of discontinuity of the normal displacements, we arrive at the static boundary-value problem of the theory of elasticity with the following boundary conditions:

$$\sigma_\theta = \tau_{r\theta} = 0, \qquad \theta = -\alpha \cup \theta = 2\pi - \alpha;$$

$$\langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \qquad \langle u_r \rangle = \langle u_\theta \rangle = 0, \qquad \theta = 0;$$

$$\langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \qquad \langle u_r \rangle = 0, \qquad \theta = \beta;$$

$$\sigma_\theta(r,\beta) = \sigma_1, \qquad \theta = \beta, \qquad r < l; \qquad \langle u_\theta \rangle = 0, \qquad \theta = \beta, \qquad r > l;$$