

Calculation of turning rigidity of I-section reinforced concrete elements with normal cracks

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Abstract. The article provides a methodology for calculating the torsion of an I-beam element using its approximation in the form of a cross-rod system. The cross-sectional I-section is divided into a finite number of longitudinal rods of rectangular section. These rods focus on their axial, flexural and torsional stiffness. Longitudinal rods are connected by transverse vertical and horizontal rods. The transverse rods imitate the local deformation of the longitudinal rods at their contact with each other. In contrast to the previously proposed methodology, in order to reduce the number of unknowns for creating the basic scheme of a statically indeterminate system, it is proposed that not all transverse rods be cut, but only one step in the longitudinal direction. The forces in the longitudinal rods determined from this calculation are applied to the rods of the next step. The pitch of the transverse rods may be arbitrary. A method for determining the stiffness parameters of the rods of an approximating system is shown. The advantages of the proposed methodology are shown, which can significantly reduce the number of equations unknown in the system, calculate elements for bending with torsion, and also take into account the change in stiffness as a result of nonlinear work of materials and crack formation.

1. Problem statement and research analysis.

A significant effect of cracks on the stiffness of reinforced concrete core elements during torsion was shown in [1,2,5,6,7]. And torsional rigidity, in turn, affects the redistribution of forces in complex systems [8,9,11]. Most torsion studies do not consider elements with normal cracks [1,2,5,6,7,12,13]. The problem of determining the torsional stiffness of reinforced concrete elements with normal cracks is quite complex. This is due to the fact that the use of formulas of the theory of elasticity is not possible due to the fact that shear stresses are not distributed over the entire cross section of the rod [8,9].

Approximate methods for calculating the strains of such elements under torsion were proposed in [3,8]. In these works, it was shown that the main task in this case is to determine the mutual displacement of the blocks of the reinforced concrete element separated by a normal crack. However, these works are notable for their approximation, especially in the case of a non-rectangular section of a rod subject to torsion.

In [3], a method was proposed for calculating reinforced concrete elements with normal cracks using a rod approximation. The disadvantage of this technique is the large number of unknowns in solving the problem. In addition, the scheme for determining the stiffness parameters of a rod

approximating system is not described there. In this work, it is shown that such an approximation is also possible for calculating I-beam elements, however, only the principles of calculating I-beam elements are given.

The purpose of this article is to develop the method of rod approximation for calculating the stiffness of reinforced concrete I-beams with normal cracks.

2. Statement of the main material.

For the calculation, we will use the rod approximation of a solid body based on the technique described in [3]. On a block length l_{cr} of an I-beam reinforced concrete element separated by normal cracks, mutually opposite torques act on two ends. Therefore, we can consider the block diagram with the length $l_{cr}/2$, sealed with one (distant) end (Figure 1).

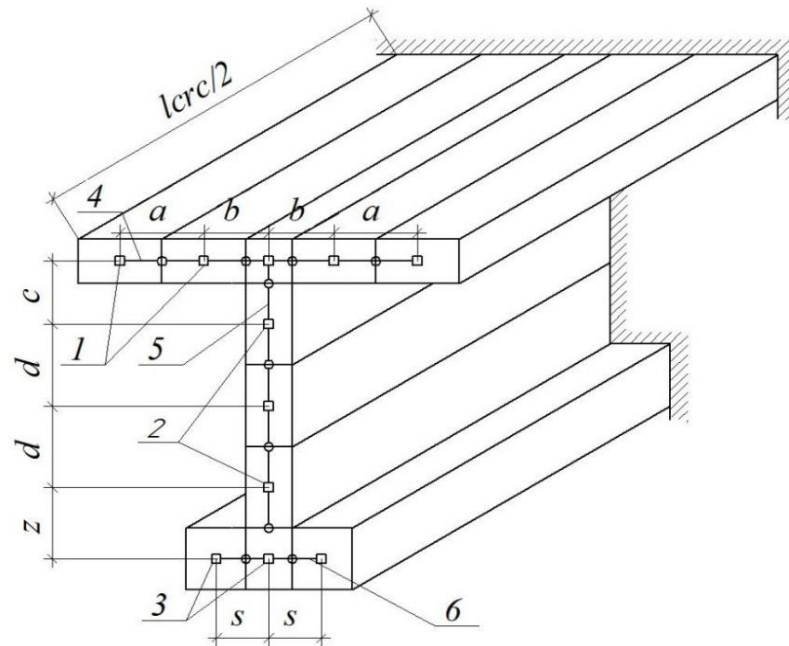


Figure 1. Scheme of an I-section element approximation by a cross-rod system.

According to the methodology [3], we divide the cross section into a finite number of rectangles. At the same time, each element of the I-beam (upper shelf, rib and lower shelf) can be divided into rectangles with arbitrary sizes.

We assume that all rectangular rods into which the I-section is conventionally divided are interconnected in the middle of horizontal and vertical faces. Thus, instead of a solid block of an I-beam cross-section, we get a system of longitudinal and transverse (vertical and horizontal) rods (Figure 2 and 3).

The number and sizes of longitudinal rods (pos. 1, 2 and 3 in Figure 1-3) are chosen arbitrarily at the discretion of the calculator. The axial, bending, and torsional stiffnesses of the rods are concentrated in the longitudinal rods of this system, into which the shelves and the rib of the I-section are divided. Transverse horizontal (pos. 4 in Figure 2, pos. 6 in Figure 3, b) and vertical (pos. 5 in Figure 3) rods simulate local deformation of longitudinal rods in the places of horizontal and vertical nodes of the longitudinal rods connection.

To significantly reduce the number of unknowns, it is proposed to dissect the transverse rods in only one section in turn. First, the first block of the cross-rod system with a length l equal to one step of the transverse rods is considered. (Figure 4). Thus, the end section with the applied external load M_r is initially considered. The longitudinal rods of the cross-rod system are considered conditionally rigidly embedded in the plane where the next row of transverse rods is located (at a distance l from the end).

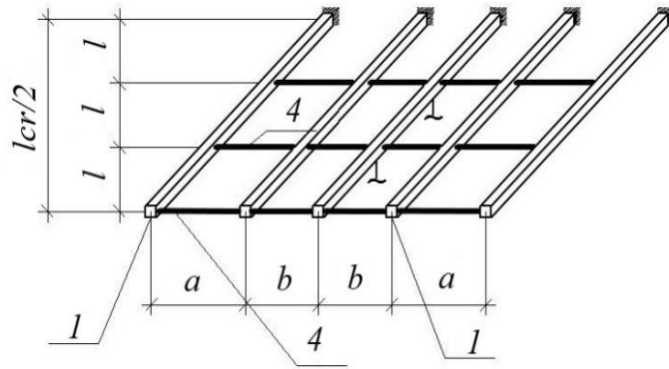


Figure 2. Representation of the upper shelf in the form of longitudinal and transverse rods.

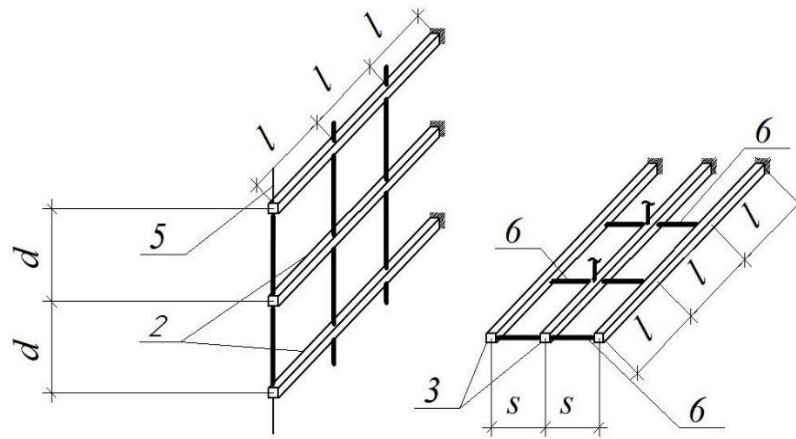


Figure 3. Representation of the ribs (a) and the lower flange (b) in the form of longitudinal and transverse rods.

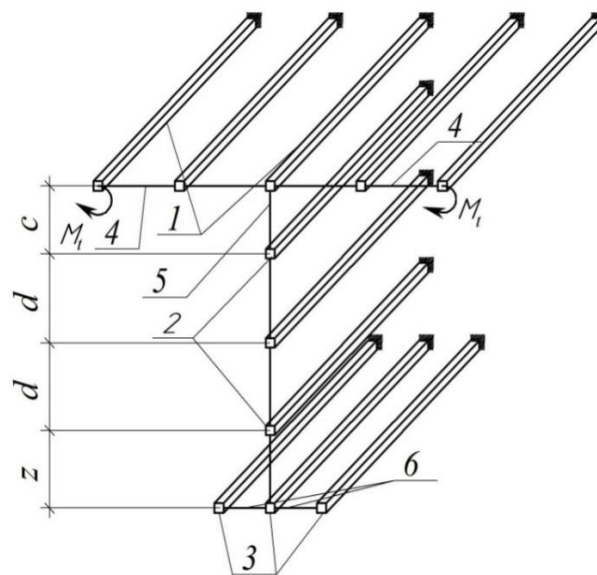


Figure 4. The core scheme for calculating the first stage

The determination of forces in longitudinal rods is carried out by conditionally dissecting the transverse rods and examining the conditions for the compatibility of deformations at the dissection sites (this technique will be described below).

The efforts in the longitudinal rods of the first block, determined from the solution of the first stage of the problem, are applied to the ends of the longitudinal rods of the second block (at a distance l from the end), which in turn are considered conditionally fixed at the level of the beginning of the third block (at a distance of $2l$ from the end). Again, the forces in the longitudinal rods are determined, which are applied to the rods of the next block, etc. This calculation is repeated until the middle of the length of the calculated block between the cracks, i.e. up to $l_{crc}/2$ as many times as the steps l in the longitudinal direction are selected in the design scheme.

To determine the displacements in the level of the i -th block along the length, add all the displacements of the blocks from the first (from the middle of the parallelepiped under consideration) to the i -1th plus the displacements of the i -th block itself.

Such a calculation scheme allows us to solve a problem with the number of unknowns n times smaller than the solution by considering the entire core system presented in [3] (where n is the number of calculation blocks along the length of the element). In this case, however, the system of equations is solved n times, but, as you know, solving a system with $m \times n$ unknowns is more complicated than solving a system with m unknown n times. In addition, with such a phased solution, it is possible to easily solve the problem with modified stiffnesses of the longitudinal rods as a result of nonlinear work of materials and crack formation.

We now proceed to determine the forces in the rods of one block under consideration (see Figure 4). To create the basic scheme of calculation by the force method, we cut all the transverse (horizontal and vertical) rods in the middle (i.e., at the places of conditionally touching the middle of the faces of the longitudinal rods). In the general case, six unknown internal forces will act at the dissection site: concentrated forces in the direction of the three coordinate axes and moments relative to the three coordinate axes. In many cases, it is sufficient to take into account only unknown concentrated internal forces in the direction of the three coordinate axes. Then the statically indefinable system will turn into a statically determinate system of longitudinal rods, to which external forces are applied, and in the places of transverse rod dissection, internal unknown forces. In figure 5 shows internal unknown forces in section j of two adjacent longitudinal rods with numbers i and $i + 1$. Moreover, in figure 5, it is assumed that the height of the cross section of the longitudinal rod is c , and its width is a . To the left of the j -th section is the i -th longitudinal rod, to the right - $i + 1$ -th. To determine the unknowns, it is necessary to draw up equations of compatibility of movements in the direction of the three coordinate axes in the considered section. Those the displacements to the left (in Figure 5) of the j -th section in the longitudinal rod with number i and the transverse horizontal rod to the right of the i -th longitudinal rod should be equal to the displacements to the right of this section in the $i + 1$ -th longitudinal rod and horizontal transverse rod of this item. Similarly, the condition of compatibility of deformations in a vertical section between two longitudinally adjacent longitudinal rods is considered.

Longitudinal rods, as mentioned above, have axial, bending, and torsional stiffnesses equal to the corresponding stiffnesses of rods of rectangular section into which the calculated double tee section is conventionally divided (see Figure 1).

Let us now consider the methodology for determining the stiffness of transverse rods imitating local longitudinal strains. Since the initial diagram assumes that all longitudinal rods are connected at points in the middle of the vertical and horizontal faces, local deformations from the acting internal forces will occur at the junction. A separate longitudinal rod, with forces acting along its right vertical face (in section j), is shown in figure 6, a . In figure 6b shows an equivalent circuit with a longitudinal rod and transverse rods.

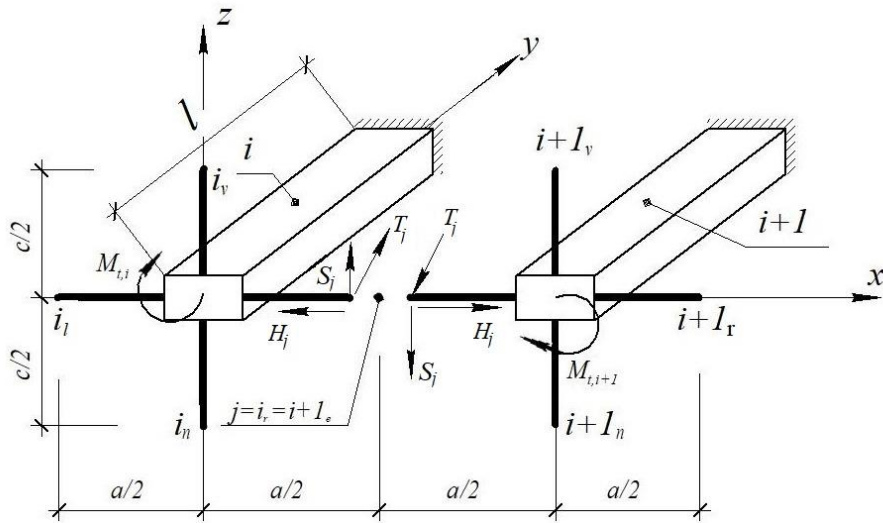


Figure 5. Unknown forces acting in section with number j

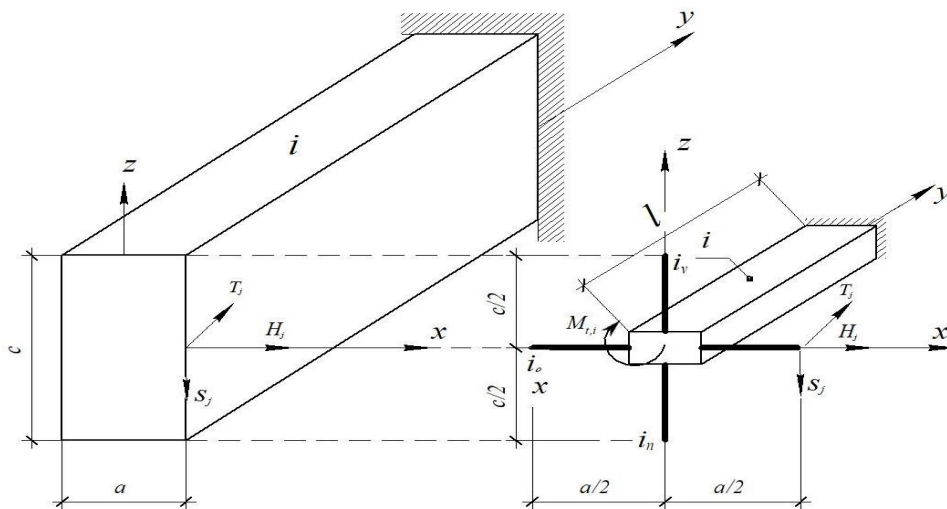


Figure 6. Scheme for determining the stiffness of the transverse rods of the calculation system

Bending and axial stiffnesses of the transverse bar on the right in figure 6b should be selected so that the real local deformation from the forces T_j, S_j and H_j in the rod in figure 6a was equal to the strain of the transverse simulating rod in Figure 6, b. Given the principle of independence of the action of forces, we can consider the deformation from each of these forces separately. For this, the approach adopted in [3] is recommended. According to this approach, a rectangular box (see Figure 6, a) should be modeled in some software package (for example, Ansys, Lira, etc.) using volumetric finite elements. Apply a unit force to such a model (separately for each case T_j, S_j and H_j) and make a series of calculations with different values of the sides of a rectangular section and different positions of forces along the length of the box. Moreover, given that when a concentrated force is applied at a point, deformations can grow unlimitedly with a decrease in the size of the finite elements, it is proposed to apply a load distributed over a certain area, the sum of which is equal to the required force H .

In figure 7 shows an example circuit for the action of horizontal forces. Variants of the application of forces along the parallelepiped are denoted by H_1, H_2, \dots, H_n . In addition, the sizes a and c of the cross section vary.

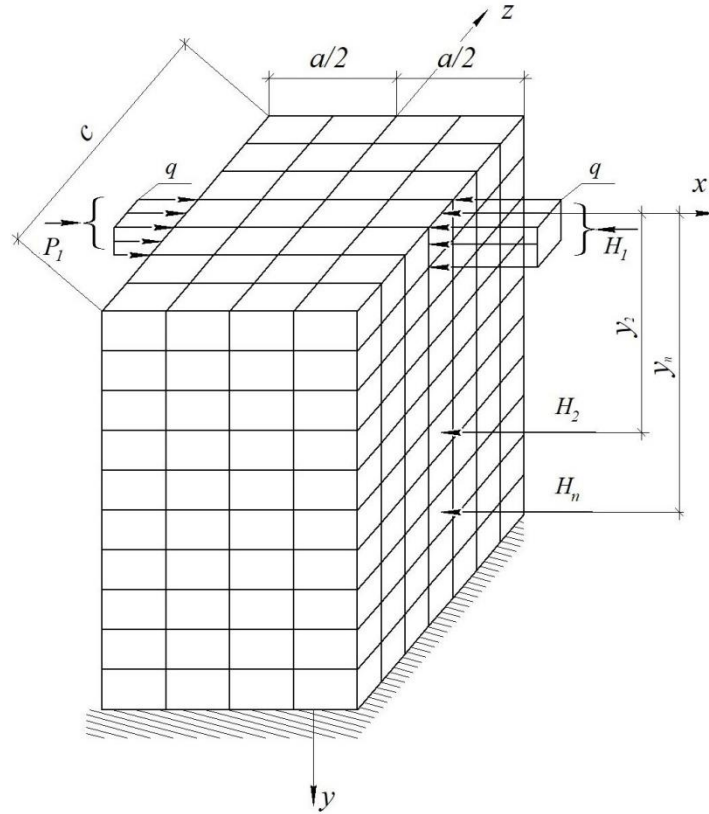


Figure 7. Scheme for determining local deformations for the case of horizontal forces

As a result of a series of calculations, a function of the form will be obtained:

$$\Delta = f(a, c, y_h) \quad (1)$$

where y_h is the distance from the end face of the box to the point of application of force along the Y axis (see Figure 7).

Once done, such a series of calculations can be further used to determine the deformations of an element of any cross section. It is easy to obtain a function similar to (1) for the action of forces S and T , shown in Figure 6.

Having Δ values for a given force, it is easy to determine the axial stiffness EA of the rod in Figure 6 b from the condition:

$$\Delta = \frac{(H-1)\frac{a}{2}}{EA} \quad (2)$$

Similarly, from the condition that the local deformation along the Y axis is equal to the action of the force T (obtained according to the scheme described above using volumetric finite element modeling) displacement of the transverse rod (see Figure 6 b) from the same force, we obtain the bending stiffness of this rod in the direction of the axis Y .

Composing for each j -th section the conditions for the compatibility of deformations to the left and right of this section (see Figure 5) and similarly above and below the section, we obtain a system of equations for determining all the internal unknown forces T_j, S_j and H_j . Given that three unknown internal forces act in each section, the system of equations will consist of $3 \cdot n$ equations with $3 \cdot n$ unknowns, where n is the number of sections.

In each i -th longitudinal rod, in the general case, there are four sections (see Figure 5): i_v above the rod, i_n in the bottom, i_l on the left, and i_r on the right. In figure 5, section j is the section i_r for the i th rod

and the section $i+l_i$ for the $i+l_i$ th rod. After determining all internal unknown forces, each longitudinal rod is calculated as a statically determinable rod, which is subjected to an external load and internal forces determined from the solution of this problem.

The following should be noted as advantages of the proposed method.

1. Having once obtained data (dependencies of type 1) based on numerical calculations of the circuit according to Figure 7, we can determine the torsion displacement of any element. In this case, the stiffness of the transverse rods of the cross-rod system (see Figure 2, 3, 6) is selected depending on the depth along the length of the element from its end, which is easy to do in a fairly simple calculation program.

2. As mentioned above, with a phased "advance" from the end of the element into its depth by considering one step l of the transverse rods, the system of equations is much smaller.

3. The proposed method of rod approximation allows you to calculate the rods of any cross section.

4. The proposed method easily solves the problem of calculating not only torsion, but also bending with torsion. To do this, not only Mt_i torques are applied at the end of the element (see Figure 5, 6), but also bending moments, as well as longitudinal forces.

5. The calculation method allows you to use any dependencies of the deformation diagram of materials, because in nonlinear calculation in each longitudinal or transverse rod, its rigidity parameters can be separately changed by any known methods for calculating reinforced concrete rod elements.

6. The technique makes it possible to rely on torsion for elements with torques distributed at their end, distributed in any way, including under the action of a moment on part of the cross section, which is practically not possible using classical methods of elasticity theory [10], in which it is assumed that shear stresses are evenly distributed over the end section.

7. The calculation procedure is simple enough to clarify by taking into account the moments relative to the coordinate axes. Moreover, in each section, six component forces should be taken into account (besides the forces considered here along the axes, there are also moments relative to the coordinate axes).

3. Сравнение с экспериментальными данными.

Применение предложенной методики покажем на примере сравнения с экспериментальными данными [9], где были испытаны железобетонные балки прямоугольного сечения с нормальными трещинами разной высоты. Перемещение $\Delta_{crс}$ в сечении с нормальной трещиной определяется по методике [4]. При этом взаимное смещение берегов нормальной трещины ΔM_t , входящее в формулу определения $\Delta_{crс}$, определялось по методике, описанной выше в настоящей статье. В таблице 1 приведены данные сравнения экспериментальных значений $\Delta = \Delta_{bloc} + \Delta_{crс}$ (где Δ_{bloc} – перемещение части блока без трещин) по [9] и теоретически с применением вышеприведенной методики для семи железобетонных образцов. Экспериментальные данные в [9] получены для балок сечением $b \times h = 120 \times 200$ мм, модуль деформаций бетона $E_c = 32097$ МПа. Диаметр арматуры d_s и высота нормальной трещины $h_{crс}$ варьируются. Теоретические и экспериментальные значения получены при значении внешнего крутящего момента $M_t = 300$ N*m. Данные таблицы 1 говорят об удовлетворительном совпадении экспериментальных данных с теоретическими, что говорит о достоверности предложенной методики расчета.

Таблица 1. Сравнение экспериментальных и теоретических значений

марка	d_s (mm)	$h_{crс}$ (mm)	Δ (mm·10 ⁻²)		$\frac{\Delta_{teor}}{\Delta_{test}}$	марка	d_s (mm)	$h_{crс}$ (mm)	$\Delta_{crс}$ (mm·10 ⁻²)		$\frac{\Delta_{teor}}{\Delta_{test}}$
			test	teor					test	teor	
B-2-1	10	150	1.65	1.59	0.963	B-4-1	10	100	1.25	1.32	1.056
B-2-2	14	150	1.45	1.51	1.041	B-4-2	14	100	1.15	1.21	1.052
B-3-1	10	125	1.59	1.72	1.081	B-4-3	18	100	1.1	1.18	1.072
B-3-3	18	125	1.23	1.31	1.065						

4. Conclusions and research prospects.

To calculate the torsion of the I-beam cross-section, it is proposed to approximate it with a cross-rod system. It is proposed to consider the sequential determination of unknowns in each row of transverse rods in order to reduce the number of unknowns. The advantages of the technique are the possibility of calculating an element of any cross section and taking into account the nonlinear properties of materials, as well as the possibility of calculating both torsion and bending with torsion.

Методика расчета позволяет определять жесткость при кручении железобетонных элементов с нормальными трещинами без применения сложных программных комплексов с использованием объемных конечных элементов. В результате использования предложенной методики появляются новые возможности определения крутильной жесткости железобетонных элементов, что в свою очередь позволяет значительно более точно вычислять усилия в сложных статически неопределимых стержневых железобетонных конструкциях.

В перспективе предполагается разработка методических рекомендаций по применению предложенной методики, а также получение аппроксимационных формул типа (1), которые могут быть использованы проектировщиками и инженерами для расчета простых и сложных сечений при кручении, причем как с наличием нормальных трещин, так и без трещин.

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