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CALCULATION OF THE PLASTIC ZONE AT A CRACK TIP WITH THE USE OF THE "TRIDENT" MODEL

A. A. Kaminskii, L. A. Kipnis, and V. A. Kolmakova

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For plane-strain conditions, the initial plastic zone near the tip of a normal-rupture crack in a uniform isotropic elastoplastic body is usually modeled by two straight slip lines originating from the tip [7]. However, the results of independent experiments [3, 4] have shown that a prefracture zone is present along with lateral plastic zones directly in front of the crack. The prefracture zone is considerably shorter than the lateral zones, and the level of the stresses in it is extremely high. The process of initial plastic deformation near the end of the crack is more accurately described by the "trident" model. In accordance with the latter, one more plastic band — a Dugdale line — emanates from the crack tip in addition of the above-mentioned slip lines. The Dugdale line is appreciably shorter than the slip lines. Using Mellin's transform and the Weiner–Hopf method, we derive formulas that express the length of the plastic regions and the opening of the crack at its tip in terms of the stress-intensity factor.

1. For plane-strain conditions, we will examine the problem of the initial growth of plastic strains near the tip of a normal rupture crack in a uniform, isotropic, ideally elastoplastic body. Since the initial plastic zone is small compared to the length of the crack and all of the other dimensions of the body and since we are studying the stress–strain state near the tip of the crack, the body can be considered an infinite elastoplastic plane containing a semi-infinite crack. The well-known asymptote that is characteristic of normal rupture cracks is realized at infinity. The accompanying stress-intensity factor K_1 is assigned in accordance with the conditions of the problem.

Proceeding on the basis of experimental results that will be detailed below, we assume that the following mechanism is responsible for the initial growth of plastic strains near the crack tip. Following the localization hypothesis, now widely used in practice and confirmed by numerous experiments [7], we assume that plastic strain during the initial stage of growth is concentrated in thin layers of the material — plastic bands. At first, two straight plastic bands grow from the crack tip with an increase in the external load. These bands make the angle α with the continuation of the crack and represent slip lines. Only the shear displacement can be discontinuous on the slip lines, and the shear stress is equal to the yield point in shear τ_s. The angle α, determined from the condition of the slip-length maximum on the basis of the exact solution obtained in [13] for the corresponding static problem, is equal to roughly 72°. Studies show that the point in the plane from which the crack and slip lines originate is also a stress concentrator but is weaker than the crack tip. Thus, slip lines growing from the tip of a crack do not eliminate the stress singularity in the tip. They only weaken it compared to the classical case. After a certain period of increase in the external load, a third plastic band begins to grow from the end of the crack (as from a stress concentrator) along with the straight slips line — which continue their growth. The third band is located along the continuation of the crack and represents a Dugdale line [14]. Only the normal displacement can undergo discontinuity on a Dugdale line, and the normal stress is equal to the yield point in tension σ_s. The Dugdale line is considerably shorter than the slip lines. With the use of Mellin's integral transform and the solution constructed below for the Weiner–Hopf functional equation, it can be shown that point O (Fig. 1), corresponding to the static boundary-value problem that will be examined below as the general problem and being the point of origin of the crack, the slip lines, and the Dugdale line, is not a stress concentrator. Thus, it should not be expected that new plastic bands will grow from the crack tip. As a result, the set of three straight plastic bands emanating from point O (two slips lines, making a 72° angle with the continuation of the crack, and the appreciably shorter Dugdale line) model the initial plastic zone at the tip of a normal rupture crack (the "trident" model).

The lengths of the plastic bands are determined from the solution of the general problem by equating the stress-intensity factors at the tips of the bands to zero.

S. P. Timoshenko Institute of Mechanics, National Academy of Sciences of Ukraine, Kiev. Umanskii Pedagogical Institute, Ukraine. Translated from Prikladnaya Mekhanika, Vol. 33, No. 5, pp. 70-76, May, 1997. Original article submitted April 23, 1994.

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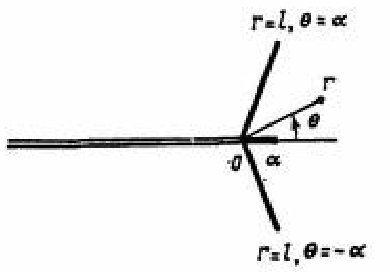

Fig. 1

In order to find the stress-intensity factor at the end of a slip line, we need only to find the solution of the problem near that end. Since the length d of the Dugdale line is considerably less than the length l of the slip lines, then for values of r commensurate with l and $d << r << l$ (in particular, near the end of one of the slip lines) we can take as the solution of the general problem the solution of the above-mentioned analogous problem without Dugdale lines. The latter is the external problem relative to the general problem. Its solution is used to find the stress-intensity factor at the end of the slip lines and the sought length. The length of the slip lines is determined by the following formula: $l = 0.0583K_I^2\tau_s^2$.

At $r \to 0$, the principal terms of the expansion of each of the stresses $\sigma_\theta(r, \theta)$, $\tau_{r\theta}(r, \theta)$, $\sigma_r(r, \theta)$ in the external problem into a series have the form $f_1^{(j)}(\theta)r^2$ and $f_2^{(j)}(\theta)$ ($j = 1, 2, 3$ corresponds to σ_θ, $\tau_{r\theta}$, and σ_r, respectively), where λ is the only root of the following equation on the interval $]-1; 0[$

$$[\sin 2(\lambda+1)\alpha + (\lambda+1)\sin 2\alpha] \times [\sin 2(\lambda+1)(\pi-\alpha) - (\lambda+1) \times \times \sin 2\alpha] + 2[\cos 2(\lambda+1)\alpha - \cos 2\alpha] \times [\sin^2(\lambda+1)(\pi-\alpha) - - (\lambda+1)^2 \sin^2\alpha] = 0 \quad (1.1)$$

(with α equal to 72°, $\lambda \approx -0.20049$). The sum of the remaining terms of each expansion approaches zero as $r \to 0$. In particular

$$\theta = 0,\ r \to 0,\ \sigma_\theta = Qr^\lambda + f(r);\quad Q = q(\alpha)\tau_s\left(\frac{K_I^2}{\tau_s^2}\right)^{-\lambda};$$

$$q = \frac{s^*}{\sqrt{\pi}}\left(\frac{\pi}{32}\right)^{-\lambda}\left(\sin\alpha\cos\frac{\alpha}{2}\right)^{-2\lambda}\frac{\Gamma(\lambda+1/2)}{\Gamma(\lambda+1)}\frac{G_0^+(-\lambda-1)}{G_0^+(-1)}\left[\frac{G_0^+(-1)}{G_0^+(-1/2)}\right]^{-2\lambda}$$

$$s^* = \frac{4(\lambda+2)\sin(\lambda+1)(\pi-\alpha)\sin\lambda\pi\sin\alpha}{\lambda s},\ s = -2\pi\sin 2\lambda\pi +$$

$$+ [2(\pi-\alpha)\cos 2\alpha - \sin 2\alpha]\sin 2(\lambda+1)(\pi-\alpha) + 4(\alpha\cos\alpha + + \sin\alpha)(\lambda+1)\cos 2(\lambda+1)\alpha\sin\alpha - [(2\alpha + \sin 2\alpha)\lambda(\lambda+2) - - (2\alpha\cos 2\alpha + \sin 2\alpha)(\lambda+1)^2]\sin 2(\lambda+1)\alpha - 4(\pi-\alpha) \times \quad (1.2)$$

$$\times (\lambda+1)\cos 2(\lambda+1)(\pi-\alpha)\sin\alpha\cos\alpha + 4(\lambda+1)\sin^2\alpha;$$

$$\exp\left[\frac{1}{2\pi i}\int_{-i\infty}^{i\infty}\frac{\ln G_0(z)}{z-p}dz\right] = \begin{cases} G_0^+(p), & \operatorname{Re} p < 0 \\ G_0^-(p), & \operatorname{Re} p > 0 \end{cases};$$

$$G_0(z) = \frac{\Delta_0(z)}{2\sin^2 z\pi},\ \Delta_0(z) = (\sin 2z\alpha + z\sin 2\alpha)[\sin 2z(\pi-\alpha) - z\sin 2\alpha] +$$

$$+ 2(\cos 2z\alpha - \cos 2\alpha)[\sin^2 z(\pi-\alpha) - z^2\sin^2\alpha],$$

($\Gamma(z)$ is the gamma function; $q(\alpha) > 0$; $f(r) \to 0$). The sums $f_1^{(j)}(\theta)r^\lambda + f_2^{(j)}(\theta)$ are the solution of problem A, analogous to the external problem with semi-infinite slip lines and corresponding to the root λ of characteristic equation (1.1).