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Calculation of reinforced concrete ceilings with normal cracks accounting the Chebyshev approximation

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Abstract. The article shows the influence of torsional rigidity of reinforced concrete elements on the spatial work of bridges, overlappings, building frames and other complex statically indeterminate systems. It is shown that the determination of torsional stiffnesses by the existing methods assumes the obligatory presence of spatial spiral cracks, and torsional stiffness in the presence of normal cracks is not investigated. A method for determining the torsional rigidity of reinforced concrete elements is described in the presence of normal cracks in them. It is shown that this approach allows calculating the torsion of reinforced concrete elements of any cross-section and taking into account the nonlinear properties of concrete. The article also describes the use of approximation methods, in particular, the apparatus of the best Chebyshev approximation. In Table 2, the displacements obtained as a result of the approximation with the displacements obtained directly from the calculations using the Lira software using volumetric finite elements are compared. In column 6 of the table, the displacement values obtained by software package (Lira software) and in column 7, the displacements obtained on the basis of approximation in the Matlab environment are given.

It is known that the torsional rigidity of their elements exerts a significant influence on the spatial work of the plate-ribbed systems. In reinforced concrete slab-ribbed systems (bridges, ribbed monolithic and prefabricated ceilings), flexural and torsional rigidity is affected by various cracks [1,2,3,4].

When local loads are applied to prefabricated or monolithic reinforced concrete floors, cracks may appear in individual beams or slabs, in others they may be absent. In this case, the torsional and flexural rigidity in the beams without cracks and with cracks will differ. It was shown in [2,3,4] that the redistribution of the local load depends in practically the same manner on both the flexural and torsional stiffnesses of the individual elements. This dependence is significant. Consequently, the definition of bending and torsion stiffness is an important and urgent task. Despite this, most calculations for the design of various structures, including well-known powerful software complexes, such as ANSYS, NASTRAN, LIRA,

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are carried out without taking into account the change in torsional stiffness resulting from the formation of normal cracks.

The torsional stiffness of individual elements with normal cracks can be taken into account in the program complexes mentioned above, when using volumetric finite elements. However, when calculating, for example, the skeleton of a multi-store building, a bridge structure or an overlap consisting of many elements, modeling [4,5,6].

Each element (column, beam, plate, etc.) from volumetric finite elements with the inclusion of reinforcement elements is very, very laborious and practically impracticable.

To date, there are a lot of studies on the flexural rigidity of reinforced concrete elements with cracks and a very small number of studies of the torsional stiffness of such elements. Most of the studies relating to torsion in reinforced concrete are devoted to the study of the strength of such elements. The existing methods for determining torsional stiffness relate mainly to reinforced concrete elements with spiral cracks under the action of torsional bending, although experimental studies have established a significant effect of normal cracks on the torsional stiffness of reinforced concrete elements. In addition, in these works we consider simple types of sections: a rectangle with a symmetric reinforcement, a ring, and cylindrical elements. Considering the fact that in practice there are a variety of types of sections: T-waves, I-beams, hollow triangles, box-shaped, etc., some studies have been carried out on this topic at the Odessa State Academy of Construction [7,8]. However, these works are only at the initial stage of research. The works devoted to the investigation of the torsional stiffness of reinforced concrete elements with normal cracks [7,8] have an approximate and particular character.

The objective of this article is developing methods for determining the torsional stiffnesses of reinforced concrete elements with normal cracks using algorithms and software complexes for processing and compressing arrays of experimental data by replacing them with a certain analytical expression (approximant) with few coefficient parameters using the known approximation methods.

Qualitatively new approach when performing such a replacement is the use of intellectualized methods of approximation of the function by the best Chebyshev (uniform) approximation, which is much more effective and universal than the interpolation and rms approximation methods.

The main advantage of the Chebyshev method of approximation in comparison with other methods of approximation is to ensure the accuracy of the approximation obtained at a certain set of points of the approximation interval at all points of this interval.

The advantages of the Chebyshev approximation allow us to solve with high accuracy not only the obtaining of an approximant and, as a result, the compression with large (several orders of magnitude) data compression coefficients of a discretely given functional dependence (the direct approximation problem), but also the task of restoring the values of the dependence on "unlit" (Inverse approximation problem) (for more details, see [9,10]).

In the works of the authors of [8], a series of studies of the torsional stiffness of reinforced concrete elements of rectangular and T-sections with normal cracks was carried out. In these works it was shown that the main part of the problem of determining the torsional stiffness of a reinforced concrete element with normal cracks is the determination of mutual displacement of the crack edges. This problem can be solved using both the approximate method and the finite element method. One of the disadvantages of solving this problem, as mentioned above, is the condition of using a large number of volumetric finite elements, which complicates both the creation of the calculation scheme and analysis of the calculation results, especially since this is only part of the solution of the general problem of the stress-strain state of a reinforced concrete element with normal cracks in torsion [4]. At the same time, the use of the methods of elasticity theory is probably not possible for all cases of solving the problem.

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Following [7], it can be assumed that the mutual displacement of crack edges will be a clear function of the height of the compressed zone, the height and width of the beam section and the distance between the cracks, which allows us to apply the methods of approximation to determine this dependence. Let us first consider the dependence of the displacements in a beam of a rectangular cross-section on two parameters-the width b and the section height h-for a fixed value of the distance between the cracks and the height of the zone compressed from the bend. In this case, perhaps not for all cases, it is possible to construct a graph of such a dependence in the form of a certain surface.

Below are examples to explain the derivation of the approximation dependencies.

Example 1. Let there be a beam of a rectangular cross-section made of a material with a modulus of elasticity E=32500 MPa, a shear modulus of 10000 MPa. The remaining parameters have such values: distance between cracks l_{crc}=400 mm; height of the compressed zone x=30 mm, torque T=1000 N·cm.

Approximation of the required surface in the Matlab environment is quite simple. The authors have previously carried out a series of calculations on the program complexes using volumetric finite elements. Variable parameters and movements are shown in Table 1.

Table 1. Variable parameters and movements between cracks.

#	Dist. between cracks l_{crc}, mm	Height of the compressed zone x, mm	b, mm	h, mm	Displacement of the crack edges (according to program complexes) $\Delta \cdot 10^3$ (mm)
1	400	30	60	100	3.104
2			80	100	1.668
3			100	100	1.043

The approximate surface obtained by processing the initial data in the Matlab environment is shown in Fig. 1.

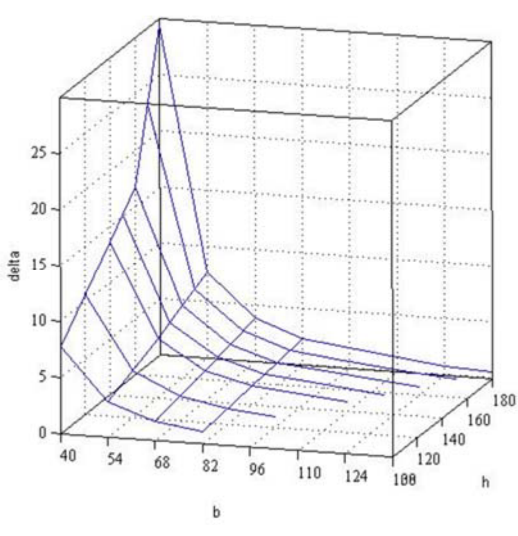

Fig. 1. The dependence of the displacement of the crack banks (delta) on Width (b) and height (h) of the beam section.

In Table 2, the displacements obtained as a result of the approximation with the displacements obtained directly from the calculations using the Lira software using volumetric finite elements are compared. In column 6 of the table, the displacements values obtained by software package (the Lira software) are given, and in column 7 the displacements obtained on the basis of approximation in the Matlab environment. As can be seen from the table, the values in columns 6 and 7 coincide with a sufficiently high accuracy, which confirms the correctness of the adopted approach to solving the problem.

Table 2. Comparison of displacements of crack edges obtained by the Lira software using volumetric finite elements and the approximation method.

#	Dist. between cracks l_{crc} (mm)	Height of the compressed zone x (mm)	b (mm)	h, (mm)	Displacement $\Delta\cdot10^3$ (mm)		Error (%)
					Finite element method	Approximation	
1	2	3	4	5	6	7	8
1	400	30	60	140	5.2469	5.3575	2.11
2			80	140	2.72018	2.7735	1.96
3			100	140	1.641188	1.6743	2.02

A similar approach is fairly simple to apply for elements with any other shape of sections (T-shaped, I-beams, box-like, etc.), and also with inclined cracks. The number of variables for the approximation can be different.

Creating a library of function approximants (similar to Table 2, Fig. 1) will greatly simplify the solution of many problems of determining the stiffness parameters of reinforced concrete elements with cracks that can enter as a separate block into existing software systems.

When determining the stiffness parameters of a reinforced concrete element with normal (or inclined) torsion cracks, to determine the movements of any points, it is necessary to compile approximate expressions (based on a certain number of calculations using finite element method), in which the cross-sectional dimensions b and h, the height of the compressed zone (The height of the zone through which the torque is transmitted), as well as the length of the block separated by normal cracks, the angle of the fracture (in the case of inclined cracks), etc.

For more complex sections, the number of variable parameters will be larger. For example, when the height of the compressed zone is within the edge of the reinforced concrete element of the I-section (Fig. 2), the displacement of the crack sides (the angle of mutual rotation of the two blocks separated by a normal crack) will be a function of seven variables:

$$\Delta crc = f(b_1 h_1, b_2, h_2, t, h, x) \tag{1}$$

To approximate such functions of several variables, one can use a software implementation of one of the algorithms of the apparatus of the best Chebyshev approximation, described in [9,10].

It should be noted that the database of values of these parameters of approximants can be obtained for specific values of the elastic modulus E and the shear modulus G of concrete. Taking into account that this problem is linear, to obtain displacements in an element with other values of elasticity and shear module, it is easy to multiply them by the ratio of the corresponding values of the parameters of the considered construction (or the corresponding iteration stage) and the values of the parameters given in the database.

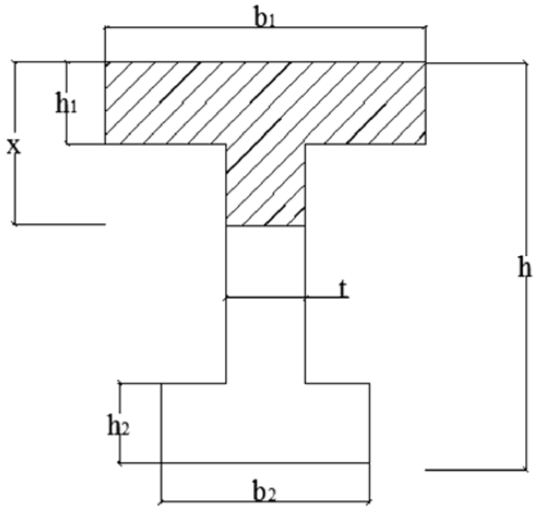

Fig. 2. Scheme to determine the parameters of the approximant for a reinforced concrete I-section with a normal crack.

Example 2. Consider the concrete case of calculating the torsional stiffness of hollow-core slabs according to the proposed technique. The section of a hollow-core plate can be represented with sufficient accuracy in the form shown in Fig. 3.

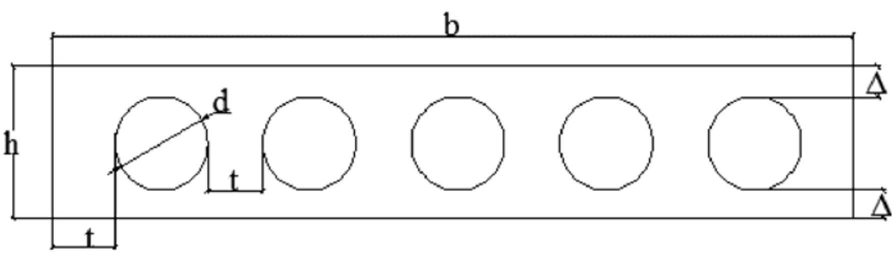

Fig. 3. Scheme to determine the approximation parameters for a hollow-core plate with a normal crack.

The function of mutual displacement of the crack banks in this case will have the form:

$$\Delta_{crc} = f(b,h,d,t,\Delta,n) \quad (2)$$

where n is the number of voids; The remaining notations are shown in Fig. 3.

In order to approximate the numerical experiment data in this case, it is effective to use the apparatus of the best uniform (Chebyshev) approximation using generalized polynomials [9,10]. Thus, for example, using this apparatus for a T-element with a width of the upper flange $b1$, thickness h_1, width of the edge b_2, height of the edge h_2, based on a series of Lira calculations using volumetric finite elements, the result of a numerical experiment for the angle of mutual rotation φ of two blocks separated by a normal crack in the form:

$$\varphi = \frac{0.00011792 b_1 h_1 + 0.1886524}{b_2 h_2 + 16.95905} \quad (3)$$

Formula (3) makes it possible to calculate the angle of mutual rotation of the crack edges for any values of the geometric parameters of the T-element. It should be noted that this formula was obtained on the basis of data from a numerical experiment with varying the geometric data of the T-section at a specific height of the compressed zone (in this case it is the thickness of the shelf of the brand). If the height of the compressed zone also needs to change, then the rotation angle function φ will contain not four variables (as in formula (3)), but five, including the height of the compressed zone (the zone through which torque is transmitted from block to block). But in any case, once obtained (although based on a fairly complex set of data from a numerical experiment), this function can be used in the design practice as many times as you like.

It should be noted that the advantages of the above method of determining the torsional stiffness by creating a database of approximant parameter values is the possibility to take into account the nonlinear properties of concrete. Since the calculation taking into account the nonlinear properties of materials is carried out using iterations, then new deformation characteristics should be adopted at each step of iterations. In the database, the mutual displacement of the shores of the normal crack is a function of the geometric parameters and the height of the zone compressed from the bending (normal crack height) of the expression (1) obtained for specific fixed values of the elastic modulus E and the shear modulus G of the concrete. Therefore, having experimental data of these parameters for a particular case under consideration, at each step of the iteration we use an expression of the type (1) with correction for the value of the elasticity and shear module at the current iteration to these parameters accepted in the database.

Creating a library of approximants will greatly simplify the solution of many complex problems, where the number of such elements would be much smaller than the number of finite elements using the traditional finite element method.

A new approach to the determination of torsional stiffnesses of reinforced concrete elements with cracks is proposed, which allows solving torsion problems of reinforced concrete elements of any cross-section using methods for approximating the displacement functions at the crack location obtained from solving a number of problems using the finite element method. It should be noted that the use of the best Chebyshev approximation in this apparatus makes it possible to improve significantly the efficiency of solving problems. Also the significant effect of torsional stiffnesses on the spatial work of bridge elements and overlapping under the action of local band loads is shown.

In the long term, it is proposed to define functions of type (2) or (3) for solving the problems of reinforced concrete elements torsion of different cross sections and their various sizes, as well as extending the proposed approach for the calculation taking into account the nonlinear properties of reinforced concrete.

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