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ANALYSIS OF PLASTIC SLIP LINES AT THE TIP OF A CRACK TERMINATING AT THE INTERFACE OF DIFFERENT MEDIA

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The slip lines at the tip of a mode I crack are analyzed by using the Wiener–Hopf technique within the scope of a plane (plane-strain) static problem of elastic theory. The crack terminates at the interface with a corner point between two isotropic media. The slip lines are located at the interface. They simulate the plastic zone near the crack tip in a piecewise-homogeneous quasibrittle body in the case where the contacting materials are much stiffer than the more plastic bonding material.

Let us consider a plane static problem on the plastic zone near the tip of a mode I crack in a piecewise-homogeneous isotropic quasibrittle body. The crack terminates at the interface between two media. The interface has a corner point. The contacting materials are assumed much stiffer than the more plastic bonding material. With such insufficient adhesion strength of the bonding material (glue) compared with the strength of the contacting bodies, transverse shear will force the plastic zones near the corner point—a stress concentrator—to extend along the interface in the form of narrow plastic bands—slip lines. On a slip line, only the tangential displacement may discontinue, and the tangential stress is equal to the shear yield point of the bonding material. Similar problems for a homogeneous body were addressed in many studies [4, 5, 8].

Since a slip line is far shorter than the crack and all other dimensions of the body, and the stress–strain state is analyzed only near those lines, we arrive at a symmetric elastic problem for a piecewise-homogeneous plane with an interface in the form of a rectilinear sides of an angle. The plane contains a semiinfinite crack and two finite slip lines at the interface, all of them emanating from the apex (problem A). In problem A, the leading terms of expansions of stresses into asymptotic series coincide at infinity with the leading terms of expansions of stresses into asymptotic series about the corner point in an external problem and are the solution of problem B (similar to problem A, but without slip lines), generated by the minimum (on the interval]–1; 0[) root of its characteristic equation. Problem B has been solved in [2]. Its solution, mentioned above, is determined to within an arbitrary constant, which is assumed given. This constant characterizes the intensity of the external field and is found from the solution of the external problem. According to Cherepanov's terminology, problem A is a problem of class N [7].

When the parameters of problem B take on values for which its characteristic equation has no roots within the interval]–1; 0[(such cases are possible, as shown in [2]), the corner point of the initial, purely elastic problem is not a stress concentrator and, consequently, will hardly ever give rise to a plastic zone in the form of narrow plastic bands under arbitrarily small external loads. Therefore, we will assume below that the parameters of problem B take on only values for which its characteristic equation has roots within the interval]–1; 0[.

Problem B for a smooth interface was studied in [12].

The boundary conditions of problem A are (Fig. 1)

$$\theta = 0, \quad \langle \sigma_\theta \rangle = \langle \tau_{r\theta} \rangle = 0, \quad \langle u_\theta \rangle = 0,$$

$$\theta = \pi - \alpha, \quad \sigma_\theta = 0, \quad \tau_{r\theta} = 0,$$

$$\theta = -\alpha, \quad \tau_{r\theta} = 0, \quad u_\theta = 0, \tag{1}$$

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