

# Model of the structure of the near tip area of interface crack in a piece-homogeneous elastic-plastic body

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**Abstract.** The model of interface crack, which assumes the existence near its tip the area of the contact of faces interactive by the law of dry friction, and the plastic zone with the small scale destruction zone in the part adjoining the crack tip with the high level of tensile and shear deformations is extended in the article. Modelling a plastic zone under the plane strain conditions by the line of displacement rupture inclined to the interface and using the Wiener-Hopf method the equations for calculating the length and orientation of the plastic zone, the sizes of the contact faces area and of the destruction zone, the stress singularity indexes near the crack tip under the conditions of the prevailing shear or tensile loading were obtained. It is established that due to the formation of a lateral plastic zone near the crack tip at the prevailing tensile loading, the length of the contact area depends not only on the configuration but also on the module of loading. Herewith its value appears to be on a few more orders than if a plastic zone was absent. The change of the stress singularity index at approach to the crack tip is revealed.

Keywords: Interface crack, contact, friction, plastic zone, destruction zone, Wiener-Hopf method

## 1. Introduction

A crack on the interface of different materials is a widespread defect in the structure of composites, glued and welded joints, covering, etc. The crack propagation between the parts of products and constructions with different physical-mechanical characteristics (piece-homogeneous bodies) in the process of exploitation is able to result in their partial or complete fracture. The necessity of the prognostication of strength, ensuring reliability and durability of the use of machine parts and elements of constructions, made of piece-homogeneous materials, cause the lasting and deep interest of research workers to the problem of interface cracks.

The research on interface cracks within the framework of a classic model of crack-cut (A.A. Griffith), founded in the 1960s and discussed in the works by Williams [45], Cherepanov [7], Erdogan [17], England [16], Rice and Sih [40], Malyshev and Salganik [34] and others, discovered physically uncorrected spatial oscillations of displacements of crack faces with the overlapping at approaching to the crack tip. The alternative model of an interface crack with the contact of part of the faces adjacent to the tip, was developed afterwards by Comninou [12,13], Comninou and Schmueser [11], Comninou and Dundurs

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[10], Gautesen and Dundurs [19], and later by Antipov [2], Loboda [32], Ostryk [36] and others, allowed to avoid the above-mentioned shortcoming of a classic model, however at predominance of the tensile loading predicted extremely small sizes of contact area, less distances between atoms on which the use of continual theory of elasticity becomes improper.

Among other mechanisms of spatial oscillations removal, Rice [39] assumed the formation of a small scale plastic zone near the tip of interface crack. The plasticity influence on the stress-strain state near the tip was confirmed in the works by Kaminskii et al. [21,22]. Their research on the plastic zone in adhesive material educed only the power feature of stresses near the tip. This result, as well as conclusions of a cohesive zone model research [42,43], points to the necessity of accounting joint nonideality not only due to the plasticity of adhesion layer but also due to its elastic properties. In particular, Antipov et al. [3], modelling a soft interface layer by certain effective contact conditions with the jump of displacements, showed the presence of stresses near the tip of the crack in the condition of anti-plane shear of logarithmic singularity.

However, taking into account only one of the factors listed above does not provide a physically correct, realistic enough and complete description of an interface if there is a presence of crack in it. Therefore, the development of an interface crack complex model, which takes into account the most substantial factors that influence the stress-strain state of piece-homogeneous body near the tip, is actual. According to the definition by Cherepanov [8], a complex of small structural elements of the area near the crack tip forms the thin structure of the tip.

The complexity of defining equations of plasticity theories does not allow us to get their exact analytical solutions. At the same time, it is desirable to know the approximate sizes of structural elements of near tip area for the numeral simulation of plastic zone taking into account the material destruction and the possibility of the crack faces contact near the tip. Such possibility is given by the approximate Leonov–Panasiuk–Dugdale model of plastic zone [14,31] which represents this zone as the line of rupture of the tangential displacement. In fact, the plastic zone area is as if cut out conditionally and her boundaries are sewn together by the corresponding boundary conditions in this model for the avoidance of the necessity of the plasticity theory nonlinear equations solution. As a result the initial problem comes to elastic theory boundary-value problem, which can be solved by the known analytical methods.

In this work the complex model of crack on the plane interface of two parts of piece-homogeneous elastic-plastic body with different mechanical properties for the plain strain conditions is developed for cases, when the sizes of contact area of the crack faces are much larger or much smaller than the sizes of a small scale plastic zone. Such size correspondence on the initial stage of plastic zone development occurs at the predominance in the external loading of shear or tension load. For the receipt of exact analytical solution a plastic zone is described within the framework of Leonov–Panasiuk–Dugdale model. In accordance with experimental data [23,24,29], it is foreseen that in the near tip part of plastic zone a very small area of material destruction with the high level of both shear and tension deformation is formed.

## **2. A model of interface crack in the prevailing shear loading conditions**

Research by Comninou and Schmueser [11], Comninou and Dundurs [10], Gautesen and Dundurs [19] and others educed that at the predominance a shear external loading the faces of internal crack on the plane interface of division of two different homogeneous isotropic materials near one of the tips have a considerable area of faces contact and far less contact near the area of an opposite tip. In this part of the